Math Problem Solving: Word Problems and Higher Math

Virtually all levels of mathematics depend on problem solving skills. Learning to solve problems involves:

- actively investigating and exploring math concepts within a word problem or math activity
- applying previously learned information to new and different situations
- forming pictures in one's mind to represent math concepts and visualize word problems
- applying the process of estimation to solving math problems
- establishing the reasonableness of a math solution

Effective math problem solving requires students to be both systematic in their approach to problems and flexible in their use of strategies.

Throughout the process of mathematical development, students are expected to operate on an increasingly abstract symbolic level. Areas of higher math, such as probability, statistics, geometry and algebra, require students to apply logical reasoning skills which are both sequential (as in multi-step equations) and spatial (as in geometric relationships).

This chart describes important skills related to math problem solving and higher math.

Necessary SubSkills	**Common Obstacles**	**Helpful Tips**
Student can use mental imagery to conceptualize math activities and can create picture of a math word problem in his mind	Student has difficulty using mental pictures (such as patterns or shapes) to represent math concepts, or has difficulty "seeing" the math problem in his/her mind.	view
Student understands math word problems he/she reads, even when irrelevant information is given, or when information is given in an order different from the order of the computation.	Student is confused by the language of word problems, such as when irrelevant information is included or when information is given out of sequence	view
Student has a systematic way of approaching math problems, and is able to break complex problems down into manageable steps.	Student doesn't know how to get started on word problems, or how to break problems down into simpler subproblems	view
Student is able to actively think through a problem, employing techniques such as applying known strategies to novel problems or adjusting strategies while working through a problem	Student has difficulty reasoning through a problem, or difficulty using strategies effectively during problem solving	view
Student works on a developmentally	Student has difficulty in grade level	view

appropriate level in areas of higher math, such as algebra, geometry, and statistics	areas of higher math, such as algebra, geometry, or statistics	

Mental Imagery/Impact of Attention and Spatial Ordering

Mental imagery refers to the process of picturing an event, concept, or procedure in your mind.

In math, one way students use mental images is to reinforce their understanding of new concepts. By translating a verbal description of a new concept into a visual picture, a student can better "see" the mathematical relationships, and create an association that improves comprehension. A student's ability to effectively move between verbal instruction and visual representations in math depends upon skills in attention and spatial abilities. Students must be able to sustain focus on details, shift between words and pictures, and interpret and organize spatial relationships by linking new instruction to existing knowledge.

Students also use mental images when solving mathematical problems. During problem solving, students must actively create visual images in their minds to represent the components of the problem. This process of visualization involves the ability to preview; that is, to form an image of an event or outcome before it occurs, e.g., to imagine what will happen when two cups of water are combined into a larger cup, etc.

Here are some strategies to help students develop their use of mental imagery in problem solving.

Helpful Hints

- Have students draw pictures to represent what is going on in a word problem. Students may draw actual objects from the problem (e.g., 3 shirts, a 6' by 12' garden plot, etc.), or they may represent objects with check marks or dots.
- Engage students' imaginations by proposing a number sentence, e.g., 6 +4 or 5(12 X 5), and having them come up with a story problem for that number sentence.
- Incorporate problem solving activities using maps, diagrams, graphs, and tables to strengthen students' use of visual/spatial materials. For example, have students calculate the distances of trips taken by students in the class, then display this information in a graph or table format.
- Involve students in making predictions in situations where visualization can aid problem solving. For example, 'If I place three green marbles and one red marble in a bag then pull one out, what color marble am I most likely to get'"
- Help students practice manipulating images in their minds in order to solve a problem. For example, provide students with a variety of shapes made from connected squares, some of which can be folded to form an open box. Ask students to find the shapes which will make an open box. Students will need to visualize the anticipated results in order to solve the problem. Many may need to develop their ability to visualize by making cut-out models and actually doing the folding. (Adapted from Brumbaugh, Ashe, Ashe & Rock, 1997).

Complex Word Problems/Impact of Language, Attention, and Sequential Ordering

Reading a complex math problem is akin to problem solving itself, placing demands on a student's language, attention, and sequential ordering skills.

A student's ability to understand the language found in math word problems greatly influences his proficiency at solving problems. Students must incorporate semantic abilities (the knowledge of specific words and their meanings), an understanding of syntax (the effects of word order and meaning of sentences), and discourse skills (understanding language beyond the sentence level, as in textbook explanations, teacher instructions, or word problems).

Once a student understands the language of a problem, he must pull out the important details, disregard extraneous information, place the crucial information in the proper sequence, or order, etc. Only when a student is able to understand the situation to be solved will he be able to complete the problem solving process.

Here are some strategies to help students strengthen their understanding of complex word problems.

Helpful Hints

- Be sure students are comfortable with one-step word problems (problems requiring only one operation) before working with multi-step word problems (problems with multiple components and operations).
- Focus specifically on the information provided in word problems. Have students separate the necessary information (numbers, labels, etc. needed to solve the problem) from the extra information (numbers, labels, and other details not needed for the solution).
- Teach students to read for meaning, rather than searching for key words, when trying to identify the operation to use for a math word problem. For example, a student who can read a problem and restate it in his own words to help him realize that he's been asked to combine amounts or add, will have a deeper understanding than a student who looks only for a key word or phrase in the sentence (e.g., 'total', 'how many', etc.) to indicate what operation to use.
- Have students create new story problems, and reword existing problems in such a way that essential information remains the same, but is worded differently. Also, have students alter important information in a problem and talk about how the problem has been changed.
- Ask students to help you come up with topic ideas for word problems, e.g., situations related to sports, popular music groups or performers, your own school, etc. Students are more likely to be interested in topics that have relevance to their lives.
- Have students paraphrase word problems for each other. Create partner pairs where one student reads a word problem silently, then provides the necessary information to his partner so the partner can do the solution.
- Have students compare textbook word problems to real life situations. For example, a textbook math problem may read " Jill bought three CDs at $14.99 each. How much did she spend'" In a real life situation, students would want to consider other factors, such as sales tax, customer discounts, etc.

Systematic Approach/Impact of Attention, Sequential Ordering, and Higher Order Cognition

Successful problem solvers are methodical, or systematic, in their problem solving. They are as concerned with the techniques they are using as they are with obtaining the right answer. These techniques may involve reorganizing a problem into simpler terms, breaking a problem into steps, making a plan about how to proceed, determining the best way to solve a problem, pulling out key ideas, etc.

Being systematic in problem solving requires students to:

- be alert to details
- preview or predict the outcomes of their actions
- sustain their effort and be goal directed
- look at the problem in different ways before choosing the best way to solve it (inhibiting first responses when necessary)
- pace themselves and self-monitor their answers at each step

Systematic problem solving often involves "step-wisdom," knowing that the best way to solve a particular problem may be to break it up into a series of logical steps, rather than to try to solve it all at once.

A systematic approach to problem solving also involves higher order thinking skills, including thinking strategically, recognizing when a problem calls for a well-thought out solution rather than an automatic response, determining the appropriate steps when breaking down a problem, ordering the steps correctly, and monitoring progress during and after problem solving.

Here are some strategies to help students become systematic in their math problem solving.

Helpful Hints

- Help students develop "step-wisdom," the ability to know when math problems need to be broken into steps to be solved, rather than done all at once. Work with the entire class to break down sample problems. First, model a step-wise approach. Let students observe how you approach problems (verbalize your steps, explain how you think through each step, etc.). Then, have students do the step breakdown, identifying what needs to be done first, what action or operation should follow next, etc.
- When assigning math activities and projects, give these assignments one step at a time to encourage students to work in stages.
- Provide students with a set of questions they can ask themselves to "jump start" their problem solving, e.g. "What does this question remind me of?", "What am I being asked to do or find?", "What are the important facts or numbers?", etc.
- Provide students with a general strategy which can be used in many problem solving situations, for example, present the following four problem solving steps (Poyla, 1945): (1) Understand the problem, (2) Make a plan for solving the problem based on the information given, (3) Carry out the plan (4) Look back at the solution.
- Teach students about strategies they can use for organizing a word problem before attempting calculations, for example, making a graphic chart that shows the important information, using a personalized checklist of steps, etc.

- Isolate specific steps in problem solving, and have students focus on one step at a time. For example, provide word problem activities in which students identify only what the question is asking them to find, which information is necessary to answer the question, which operations should be used in the problem, or whether or not the answer provided to a word problem makes sense.
- Explain to students that good problem solvers rarely skip steps when problem solving, although it might seem that they do. Instead, problem solvers learn to do steps mentally (in their heads) instead of writing them down or talking about them. Suggest that with experience, students may learn to do this, too.

Active Solving and Strategy Use/Impact of Attention, Sequential Ordering, Memory, and Higher Order Cognition

Computation is a vital component of math. But students need to focus on more than just accuracy in calculation. The ability to reason through a word problem, to think critically about all aspects of the mathematical situation, is an increasingly important goal in math instruction.

Active problem solving skills are strongly tied to a student's attention abilities. Students must closely attend to problem details to determine what the question is, what kind of answer to look for, and what information will be salient or important when solving the problem. Students must analyze the problem, possibly breaking it down into a logical sequence of smaller steps (instead of reacting impulsively or 'jumping' to a conclusion). In addition, the ability to preview, or estimate likely outcomes within a problem, enables the student to make quantitative and strategic predictions, such as what amounts will likely be involved or what strategies are likely to be used.

Students use their long-term memory to help determine whether a word problem reflects a familiar pattern. They search in memory for prior knowledge, learned rules, or relevant skills that have worked in the past for that type of problem, and then, apply that knowledge in the new situation.

Effective problem solving also requires flexible thinking. Students may need to use, evaluate or change strategies; at times, they may need to consider several alternative strategies. And, finally, throughout the problem solving process, students must be able to monitor the outcomes of their calculations, and refine their solutions when necessary.

Here are some strategies to help students become active problem solvers in math.

Helpful Hints

- Model problems for the class and explain each step when teaching students how to be active problem solvers. Think out loud for students as you reason through a problem, choose a strategy to use, decide if the strategy is working, etc. Have students talk through problems with each other as well.
- To promote strategy use and adjustment, ask students guiding questions as they solve problems, e.g., "Is there an easier way to do that'", "Will that strategy always work'", etc.
- Have students communicate their understanding of a problem through both oral discussion and written explanation.
- Have a brainstorming session with students to discuss the types of behavior or steps are involved in problem solving, characteristics of 'good problem solvers', etc. Some

ideas may include reasoning, looking for patterns, patience, persistence, hypothesizing, stating the obvious, creativity, etc.

- Encourage students to explore multiple strategies that could be used for solving a math problem. For example, ask students to find the length of the diagonal of a 12" x 16" rectangle. Students will likely recognize that the rectangle is made up of two right triangles, and apply the Pythagorean theorem. One approach might be to calculate by hand or to use a calculator for the computation ($12^2 + 16^2 =$ ') to eventually come up with the answer of 20 inches.
- However, an alternate view of the problem makes it even easier to solve. A student might notice that the 12" and 16" sides are both divided evenly by 4, resulting in a triangle with sides of 3" x 4", respectively. The student will likely recognize the missing diagonal length to be 5", making the standard 3" x 4" x 5" triangle. Then, simply multiplying the 5" back by 4 would give the answer to the diagonal: 20 inches. No lengthy computations would be needed. (Adapted from Brumbaugh, Ashe, Ashe & Rock, 1997).
- Have students practice selecting what strategies might be appropriate for solving a given problem. For example, in each case, would it be helpful to act out the problem, make a model, draw a picture, make a chart or graph, use logic, guess and check, break it into parts, etc.'
- Promote students' flexible thinking by presenting situations in which there is more than just one right answer. For example, have students take out a piece of paper, fold the paper in half, then fold the paper in half again. Ask students to count how many rectangles have been formed. Answers will vary depending upon how the second fold was made in the paper, if students count the whole piece of paper as one of the rectangles, etc. Have students discuss how different folding approaches resulted in different 'answers'. (Adapted from Brumbaugh, Ashe, Ashe & Rock, 1997).
- Give students practice estimating the answers to problems. Have them move from estimation to calculation, then back again to estimation. Help students develop their sense, before starting calculations, of what a general solution to the problem might be, and also to take time to examine their answers, after calculation, to see if they seem credible. Students may require guidance in useful strategies for estimation, e.g., rounding numbers, creating a visual image, etc.

Higher Math/Impact of Skill Readiness

Early in math instruction, students are informally introduced to areas of higher math, such as geometry, probability, and statistics. Children experience geometry in the form of basic shapes and figures, part/whole relationships, and basic patterns. Laws of probability and chance are presented through games with cards and dice. Activities involving collecting, organizing and classifying objects provide the foundation for statistics.

Background for the study of algebra begins in the early grades as well. The simple equation 4+2 =?, for example, uses the algebraic concept of an unknown ("?") representing a quantity (in this case 6). Solving equations with fractions in middle grades represents another building block for algebra readiness.

As students move to areas of higher math, they will find that these areas are also related and interdependent. For example, core concepts in pre-calculus require background skills in advanced graphing, coordinate and space geometry, laws of probability, statistical procedures, and algebraic expressions.

The transition into the formal teaching of higher math typically occurs with the high-school core curriculum. Background material must be developed for students at each step along the way in early, middle, and even higher grades to prepare them for the more formal instruction to come.

Here are some suggestions for helping students progress in areas of higher math.

Helpful Hints

- Establish that students have the necessary background skills to move ahead to formal instruction in areas of higher math. For example, students who have not mastered factoring from Algebra I will have great difficulty simplifying rational expressions in Algebra II.
- Utilize computer software programs to help students explore areas of higher math. Programs exist for all levels and areas. Incorporate tutorial programs that are interactive and dynamic.
- Set up a "math mentor" for the student. This person may be a mathematics teacher, or a professional in the community who uses math in his/her work, e.g., a surveyor, an architect, a research scientist, an accountant, etc.
- Use real life problem solving to help students connect concepts in higher math. For example, when students are exploring the question of how a spacecraft stays in orbit around the earth they will use formulas for gravity, geometric concepts, proportion formulas, etc.