Understanding Math Concepts

For a student to progress in mathematics, several conceptual building blocks must be acquired. Such components include:

- the ability to form and use associations (as in number concepts and symbol use)
- a grasp of the language of Mathematics, from concepts such as measurement and money to the technical vocabulary of math such as *parallelogram* and *denominator*.
- an understanding of the relationships involved in numeric operations (such as the place value concept behind borrowing and carrying)
- the ability to make generalizations (as in the application of mathematical learning to everyday situations)

This chart describes important skills related to understanding math concepts:

Necessary SubSkills	Common Obstacles	Helpful Tips
Student understands mathematical symbols and can visualize patterns, math concepts, and the parts of a problem in his/her head.	Student has difficulty visualizing patterns or the parts of a math problem in his head. Student has difficulty associating math symbols with the concepts they represent.	view
Student understands math vocabulary words and is able to build math knowledge through the use of math language.	Student is not comfortable using mathematical language, or has difficulty with math vocabulary words.	view
Student understands how concepts are related (as in the relationship between addition and subtraction, or between ratio and proportion).	Student has difficulty seeing how concepts (such as addition and subtraction, or ratio and proportion) are related to each other.	view
Student can see how math concepts (such as proportion or measurement) apply to everyday life.	Student has problems transferring concepts learned in the math classroom to real life situations.	view

Symbols and Visualization/Impact of Higher Order Cognition and Spatial Ordering

In developing an awareness of mathematical concepts, students must engage their nonverbal thinking skills. Nonverbal thinking involves the use of spatial and visual processes to learn or think about a problem or concept.

Nonverbal thinking may involve the use of symbols. The numerals 6 and 26, for example, are symbols that represent quantities. Students use and manipulate symbols when doing operations ranging from basic addition to algebraic equations.

Nonverbal thinking also may involve visual or spatial representations of math processes and relationships. Students must be able to interpret visual and spatial information (as when looking at a map, graph, or geometric shape), and to form and understand visual and spatial concepts (as when translating graph images into usable mathematical information, or describing attributes of shapes).

Some concepts lend themselves to 'visualization', creating a mental image to represent a mathematical relationship. The concept of proportion is a good example. A student may have a difficult time interpreting proportion through words and verbal explanation, but being able to visualize the relationship (e.g., the number of boys to girls in the class, the ratio of eaten slices in a pizza) may greatly enhance his/her understanding of proportion as a concept.

Here are some strategies to help students develop and strengthen their understanding of symbols and their abilities to visualize.

Helpful Hints

- Integrate hands-on activities and verbal explanations into the learning of spatially based concepts. For example, have students use pattern blocks or geoboards to make geometric shapes, then discuss and write down the characteristics of the shapes, such as number of sides, types of angles, etc.
- Use examples of familiar situations, or analogies, to talk and think about math concepts. This helps students link the concepts to a visual image. For example, the concept of ratio may be illustrated by asking students to imagine two brothers sharing a pizza, and the amount of pizza left over after the big brother takes his portion.
- Guide students in visualizing patterns. For example, talk students through 'seeing' a geometric shape in their minds, 'picturing' a math process taking place, such as 1/3 of a pizza being taken away, and 2/3 of the pizza remaining, etc.

Math Vocabulary/Impact of Language, Memory, and Sequential Ordering

Mastering the language of mathematics is much like mastering a foreign language. The content is both new and cumulative, as new learning enhances abilities by complementing and expanding upon what a student already knows.

As students progress in math, they must grapple with many complex verbal explanations and a growing vocabulary of terms that are rarely used outside math situations (e.g., trapezoid and dividend). For this reason, a student's language skills and comfort with new vocabulary can have a great impact on mathematical development.

The ongoing acquisition and use of math language is also aided by a student's ability to read and listen carefully, to organize terms in memory and recall those terms as needed, to follow sequences of rules and procedures, and to use language abilities to enhance understanding of math concepts.

Here are some strategies to help develop and strengthen students' use of math language.

Helpful Hints

- Make reading about math an activity you and your class do together. Math vocabulary can be reinforced by reading biographies of mathematicians and inventors, books about the history of math, fictional stories with characters that work with math, news and sports stories involving math, etc.
- Have students use their new math vocabulary words to teach their parents, younger siblings, younger students, or peers about the concepts they have learned.
- Have students keep a personal math vocabulary book in which they record new math terms. Have them actively link the new terms to their existing knowledge by drawing pictures next to terms to have a visual representation, by showing examples that match and don't match the concept, by creating a flowchart of terms or diagram of pictures to show how a concept fits in with other math concepts, etc.

Math Relationships/Impact of Higher Order Cognition, Spatial and Sequential Ordering, and Memory

In order to understand the mathematical relationships occurring in a problem or equation, students must understand the broad concepts involved. Some math relationships are spatial, they involve physical objects and/or physical space (e.g., the relationship between an object's weight and its size or mass). Other math relationships are sequential, the order in which steps occur or elements act on each other is most important (e.g., the relationship between the equations '10x+4=24' and '10x=24-4').

To work effectively with math relationships, students must have a flexible approach to each problem, knowing that quantities may be represented in a variety of ways. For example, understanding the concept of place value (e.g., that 30 is the same as 3 tens) enables students to more easily deal with problems concerning money, just as understanding the concept of units and subdivisions (or parts and wholes) helps students divide a single candy bar into equal parts to share.

Finally, students must be able to store and retrieve concepts from long term memory, and to hold several symbols and concepts in their minds. For example, a problem requiring plotting information on a graph may involve multiple concepts, including collecting and organizing information, setting up ratios, finding averages, using the coordinate system, etc. Here are some strategies to help develop and strengthen students' understanding of relationships in math.

Helpful Hints

- Provide plenty of hands-on practice with concepts that are typically confused, such as weight and mass, capacity and volume, area and perimeter, etc.
- Use manipulatives to help students explore mathematical relationships. For example, Connecting People (available from the Cuisenaire Company), are small, connectable figurines of different colors, sexes, and sizes. Activities can be built around the Connecting People figurines in which students build patterns, use math concepts in stories, organize and classify, use estimation, collect data, and explore units of measurement. (Welchman-Tischler, 1995).

- In addition to using manipulatives and hands-on activities, have students develop charts and diagrams, or create note cards to define terms, show examples, etc. to explore how math terms are related.
- Have students use different representations to describe the same situation. For example, demonstrate how something can be shown using a table, a graph, written description, etc.
- Give students access to a dynamic or interactive computer software program that allows them to manipulate symbols, compare concepts, etc.

Concept Application/Impact of Attention, Memory, and Higher Order Cognition

An important goal of math instruction is for students to see the relevance of math concepts to everyday life. A student's progression from a basic understanding of a concept to a particular level of competency in applying that concept in real-life situations is strongly influenced by abilities in attention, memory and higher order thinking.

Attention skills help students maintain a steady focus upon the details of math concepts, as they link new concepts to what they already know about math. Memory skills help students to store and retrieve known concepts, and to elaborate on new concepts in their minds. Applying new concepts depends not only on memorization skills (for example, memorizing attributes of the concept of volume), but also upon the ability to think about these concepts in relation to real-life situations (e.g., applying the concept of volume in science class or in the kitchen at home).

Here are some strategies to help students develop and practice the application of math concepts to everyday life.

Helpful Hints

- When teaching basic concepts, let students work with concrete objects in the classroom. For example, let students explore number concepts by adding and subtracting objects in the room (e.g., finding the number four by adding the legs on two desks, by subtracting crayons from the box, etc.).
- When introducing broad concepts, give students opportunities to connect these concepts to prior experience and relevant situations. For example, reinforce measurement concepts by having students compare the height of classmates, or the weight of their book bags when empty and full. Have students first estimate measurements (e.g., how many books the bag can hold, how much taller John is than Matt); then solve exact measurements.
- Identify topics that would be of interest to your students, such as building a skateboard ramp, being a market researcher, etc., and explore the mathematical relationships or concepts related to these topics.
- Help students learn to apply math concepts to new situations. Provide specific instructions (and ongoing prompting) that describe what to look for, and the steps to follow when applying each concept. For example, teach students to use the concept of percent to examine the amount of water in the human body, the price of a jacket on sale at the mall, the portion of students who have pets, the percentage of allowance spent on entertainment, etc.
- Have students identify daily situations where they use math skills, for example, when reading bus schedules, filling out catalog order forms, etc.

- Integrate historical information and events into your discussions to connect math concepts to everyday life. For example, have students explore how the needs of the times prompted people to create or define math concepts and ideas, e.g., the need to build the pyramids, the desire to navigate the ocean, etc. Have students write a biographical portrait about a person to whom math was important, e.g., Pythagoras and his extension of the Pythagorean theorem, Florence Nightingale and her innovative use of statistics, or Federal Reserve chairman Alan Greenspan and the use of math in economics.